PLANE-PARALLEL ADVECTIVE FLOWS IN A VIBRATION FIELD

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UDC 536.25

We examine the effect of harmonic high-frequency vibrations on advective plane-parallel liquid flow in a plane horizontal layer.

A plane-parallel advective flow arises in a plane horizontal layer of a liquid in the presence of a constant longitudinal temperature gradient [1, 2]. Flows of this type are of interest, in particular, in connection with an examination of the process of growing crystals by the method of directed crystallization (see [3]). Below we consider the effect of harmonic high-frequency vibrations on advective flow.

A plane horizontal liquid layer of infinite extent in the horizontal directions is bounded by solid parallel planes x = ±h (the x axis is directed vertically upward, while the y and z axes are directed in the plane of the layer). A constant temperature gradient A is given for both boundaries of the layer, which is directed along the z axis: ∇T (0, 0, A). The liquid layer executes harmonic vibrations in some direction characterized by the unit vector **n**; Ω denotes the vibration frequency and the displacement amplitude is denoted b. The equations describing the averaged fields of velocity v, temperature T, and pressure p, in the limit case of high frequencies, have the form

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\frac{1}{\rho}\nabla p+\nu\Delta\mathbf{v}+g\beta T\boldsymbol{\gamma}+\varepsilon(\mathbf{w}\nabla)(T\mathbf{n}-\mathbf{w}),$$
$$\frac{\partial T}{\partial t}+\mathbf{v}\nabla T=\chi\Delta T,\quad \operatorname{div}\mathbf{v}=0,\quad \operatorname{div}\mathbf{w}=0,\quad \operatorname{rot}\mathbf{w}=\nabla T\times\mathbf{n}. \tag{1}$$

Here w is the solenoidal portion of the vector field T**n**, proportional to the amplitude of the pulsation velocity component; $\varepsilon = (\beta b\Omega)^2/2$ is the parameter which defines the effect of vibration on convection at the limit of high frequencies. Equations (1) are frequently utilized to solve various problems in the theory of convection when high-frequency vibrations are present [4-6].

Let us initially consider the case of a longitudinal vibration in which the vector is parallel to the temperature gradient at the boundaries: **n** (0, 0, 1). Equations (1) in this case allow for a solution which describes the plane-parallel advective flow of the following structure:

$$v_x=v_y=0,\quad v_z=v(x);\quad T=Az+\tau(x);$$
$$p=p(x,\ z);\quad w_x=w_y=0,\quad w_z=w(x). \tag{2}$$

After dropping pressure and after separation of the variables, from Eqs. (1) we obtain a system of ordinary differential equations (the prime denotes differentiation with respect to the lateral x coordinate)

$$\nu v''+\varepsilon Aw-g\beta Ax=C_1,$$
$$\chi\tau''=Av,\quad \tau-w=C_2, \tag{3}$$

where C_1 and C_2 are the constants of variable separation. We will assume the flow to be closed (the opposite ends of the layer $z \to \pm\infty$ are impermeable to the liquid); in this case, the profiles of v, τ, and w are odd functions of x, and $C_1 = C_2 = 0$. Conditions of adhesion $v(\pm h) = 0$ are set for the boundaries of the layer. As regards temperature, we will subse-

Perm State University. Perm State Pedagogical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 2, pp. 238-242, February, 1989. Original article submitted October 26, 1987.

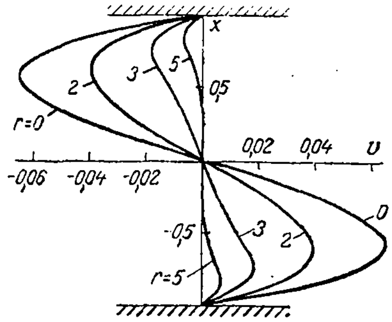

Fig. 1. Profiles of dimensionless velocity (in units of $g\beta Ah^3/\nu$) for the case of high heat-conduction boundaries.

sequently consider two boundary-layer versions: a) the boundaries exhibit high thermal conductivity and at these boundaries we maintain the temperature $T(\pm h) = Az$, i.e., $\tau(\pm h) = 0$; b) thermally insulated boundaries, i.e., $\tau'(\pm h) = 0$.

Let us present the profiles of the quantities v, τ, and w, characterizing the advective flow. In case a) the solution of the problem has the form

$$v = \frac{g\beta Ah^3}{2\nu}\frac{1}{\delta_1 r^2}\left[\frac{\operatorname{ch} r\xi \sin r\xi}{\operatorname{ch} r \sin r} - \frac{\operatorname{sh} r\xi \cos r\xi}{\operatorname{sh} r \cos r}\right], \tag{4}$$

$$\tau = w = \frac{g\beta A^2h^5}{4\nu\chi}\frac{1}{r^4}\left[\xi - \frac{1}{\delta_1}\left(\frac{\operatorname{ch} r\xi \sin r\xi}{\operatorname{sh} r \cos r} + \frac{\operatorname{sh} r\xi \cos r\xi}{\operatorname{ch} r \sin r}\right)\right], \tag{5}$$

where $\xi = x/h$ is a dimensionless lateral coordinate; $r = (\varepsilon A^2h^4/4\nu\chi)^{1/4}$ is the dimensionless vibration parameter associated with the Rayleigh vibration number through the relationship $Ra_V = 4r^4$; $\delta_1 = \tan r \coth r + \cotan r \operatorname{th} r$.

For thermally insulated boundaries (case b)) we obtain

$$v = \frac{g\beta Ah^3}{4\nu}\frac{\delta_2}{r^3}\left[\frac{\operatorname{ch} r\xi \sin r\xi}{\operatorname{ch} r \sin r} - \frac{\operatorname{sh} r\xi \cos r\xi}{\operatorname{sh} r \cos r}\right], \tag{6}$$

$$\tau = w = \frac{g\beta A^2h^5}{4\nu\chi}\frac{1}{r^4}\left[\xi - \frac{\delta_2}{2r}\left(\frac{\operatorname{ch} r\xi \sin r\xi}{\operatorname{sh} r \cos r} + \frac{\operatorname{sh} r\xi \cos r\xi}{\operatorname{ch} r \sin r}\right)\right], \tag{7}$$

where $\delta_2 = \operatorname{sh} 2r \sin 2r/(\operatorname{sh} 2r + \sin 2r)$.

The profiles of velocity (4) and temperature (5) for the case of boundaries exhibiting high heat conduction are shown in Figs. 1 and 2a. As we can see, the flow consists of two oppositely directed advective streams: in the upper portion of the layer the warmer liquid flows in a direction opposite to the longitudinal temperature gradient, while in the lower portion the cold liquid moves in the direction of the gradient. At the limit as $r \to 0$, from formulas (4) and (5) we obtain certain distributions which correspond to the absence of vibration:

$$v = \frac{g\beta Ah^3}{6\nu}(\xi^3 - \xi), \quad \tau = \frac{g\beta A^2h^5}{360\nu\chi}(3\xi^5 - 10\xi^3 + 7\xi). \tag{8}$$

With an increase in the vibration parameter r the flow is retarded and the lateral nonuniformity of the temperature field diminishes. With larger r the flow assumes the structure of noninteracting boundary layers whose thickness with increasing r diminishes as h/r.

In the case of thermally insulated boundaries [formulas (6) and (7)] the velocity profiles are similar in shape to those shown in Fig. 1, differing from these only in terms of scale that is dependent on r. The effect of flow retardation as a consequence of vibration is expressed more strongly than in the case of boundaries of high heat conduction. In the absence of vibration (r = 0), at the limit from formulas (6) and (7) we have the same velocity profile as in (8), and the temperature distribution has the form

$$\tau = \frac{g\beta A^2h^5}{360\nu\chi}(3\xi^5 - 10\xi^3 + 15\xi). \tag{9}$$

The temperature profiles (7) are shown in Fig. 2b. The significant difference from Fig. 2a lies in the formation of a relatively greater temperature difference across the boundary layers

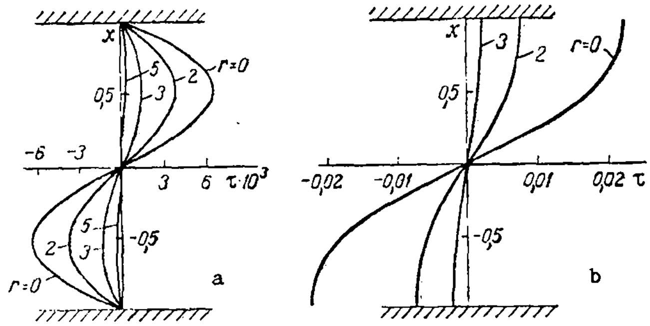

Fig. 2. Profiles of dimensionless temperature (in units of $g\beta A^2h^5/\nu\chi$) for the case of high heat-conduction boundaries (a) and for thermally insulated boundaries (b).

$$\Theta = \tau(h) - \tau(-h) = \frac{g\beta A^2h^5}{\nu\chi}\,\frac{1}{2r^4}\left[1 - \frac{4}{r}\,\frac{\mathrm{sh}^2 r + \sin^2 r}{\mathrm{sh}\,2r + \sin 2r}\right]. \tag{10}$$

Without vibration $\Theta = 2g\beta A^2h^5/45\nu\chi$; with an increase in r the temperature difference diminishes in proportion to $1/r^4$.

The pronounced retarding effect of vibration in the case of thermally insulated boundaries is understandable. We know [7] that in a layer with a longitudinal temperature gradient in the presence also of a lateral temperature difference the longitudinal high-frequency vibration itself (in the absence of a static gravity field) leads to the appearance of an averaged flow whose velocity near the heated boundary is directed along the gradient; thus, the gravitational-advective and vibrational-convective components of the flow exhibit opposing directions.

The intensity of the flow can be characterized by the flow rate of the liquid in one of the opposed flows

$$Q = \int_{-h}^{0} v\,dx. \tag{11}$$

If no vibration is present in either of these cases, $Q_0 = g\beta Ah^4/24\nu$. In the presence of vibration for cases a) and b) we have, respectively,

$$\frac{Q}{Q_0} = \frac{6}{r^3}\,\frac{\mathrm{sh}\,r - \sin r}{\mathrm{ch}\,r + \cos r}; \tag{12}$$

$$\frac{Q}{Q_0} = \frac{12}{r^4}\,\frac{(\mathrm{sh}\,r - \sin r)(\mathrm{ch}\,r - \cos r)}{\mathrm{sh}\,2r + \sin 2r}. \tag{13}$$

With an increase in r the rate of flow diminishes monotonically (Fig. 3). For large r we have an asymptote: a) $Q/Q_0 = 6/r^3$; b) $Q/Q_0 = 6/r^4$, i.e., in the case of thermally insulated boundaries the intensity of the flow is smaller and diminishes more rapidly with increasing r.

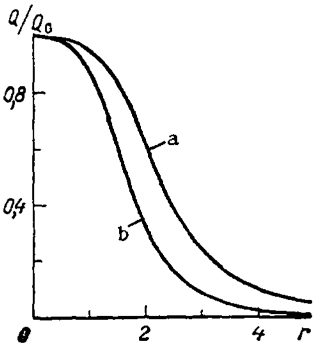

Fig. 3. Relative liquid flow rate as a function of the vibration parameter for heat-conducting (a) and thermally insulated (b) boundaries.

Let us now turn to the case of vibration transverse to the plane of the layer: **n** (1, 0, 0). In this case we have a solution of the problem, which describes the closed plane-parallel advective flow of structure (2). From the general system (1) we obtain

$$\nu v'' - g\beta A x = 0, \quad \chi\tau'' = Av, \quad w' = -A. \tag{14}$$

The first two equations of this system, together with the corresponding boundary conditions, determine the velocity and temperature profiles which, as we can see, coincide with the corresponding profiles in the absence of vibration, i.e., Eqs. (8) and (9). The transverse vibration thus has no effect on the gravitational-advective flow and leads only to the appearance of the longitudinal pulsation velocity component which corresponds to the field $w = -Ax$.

Finally, let us consider the case in which the vibration axis is horizontal and perpendicular to the temperature gradient: **n** (0, 1, 0). Unlike the previous two cases, in the plane-parallel flow regime the field w now exhibits a structure $w_x = w_z = 0$, $w_y = w(x, z)$. As in the case of transverse vibration, the profiles of velocity v and temperature τ are independent of the vibration parameter r and are determined from formulas (8) and (9). The presence of vibration leads to the appearance of a pulsation velocity component along the y axis with the field $w = Az + \tau(x)$.

NOTATION

g, acceleration of the force of gravity; ρ, mean density; β, ν, χ, coefficients of thermal expansion, kinematic viscosity, and thermal conductivity; γ, unit vector directed upward along the vertical; A, horizontal temperature gradient; r, a dimensionless vibration parameter; v, p, T, averaged fields of velocity, pressure, and temperature; w, solenoidal portion of the field T**n**.

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